9.4 Operations with Functions

Notation for Function Operations	
Operation	Notation
Addition	$(f + g)(x) = f(x) + g(x)$
Subtraction	$(f - g)(x) = f(x) - g(x)$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$
Division	$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$, where $g(x) \neq 0$

$$f(g(x)) = (f \circ g)(x)$$

The Composite of 2 functions:

- The composite of 2 functions f and g is the function $f(g(x))$, which is read: "f of g of x."
- You write $(f \circ g)(x)$, which is read "the composite of f and g."
- To find $(f \circ g)(x)$, you find g(x) first and then find f(x) for that value.

Example: $f(x) = x^2 + 1$ $g(x) = x + 3$ $h(x) = 4x - 5$

1. $(f \circ g)(3)$

$\boxed{f(g(3)) = 37}$

$g(3) = 3 + 3 = 6$

$f(6) = 6^2 + 1 = 37$

2. $(g \circ f)(3)$

$\boxed{g(f(3)) = 13}$

$f(3) = 3^2 + 1 = 10$

$g(10) = 10 + 3 = 13$

3. $(f \circ g)(x)$

$f(g(x))$

$f(x+3) = (x+3)^2 + 1$

$= x^2 + 6x + 9 + 1$

$\boxed{f(g(x)) = x^2 + 6x + 10}$

4. $(g \circ f)(x)$

$g(f(x))$

$g(x^2 + 1) = x^2 + 1 + 3$

$\boxed{g(f(x)) = x^2 + 4}$

5. $(f - g)(x)$

$f(x) - g(x)$

$x^2 + 1 - (x + 3)$

$x^2 + 1 - x - 3$

$x^2 - x - 2$

$\boxed{(f - g)(x) = x^2 - x - 2}$

6. $(f + g)(x)$

$f(x) + g(x)$

$x^2 + 1 + x + 3$

$x^2 + x + 4$

$\boxed{(f + g)(x) = x^2 + x + 4}$

$$f(x) = x^2 + 1 \qquad g(x) = x + 3 \qquad h(x) = 4x - 5$$

7. $(fg)(x)$ multiply!

$f(x) \cdot g(x)$

$(x^2 + 1)(x + 3)$

$x^3 + 3x^2 + x + 3$

$\boxed{(fg)(x) = x^3 + 3x^2 + x + 3}$

8. $\left(\dfrac{f}{g}\right)(x) \quad \dfrac{f(x)}{g(x)}$

$\dfrac{x^2 + 1}{x + 3}$

$\boxed{\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2 + 1}{x + 3}}$

9. $\boxed{(f \circ g \circ h)(5) = 325}$ ◯

$f(g(\underline{h(5)}))$

$h(5) = 4(5) - 5$

$h(5) = 15$

$g(15) = 15 + 3 = 18$

$f(18) = (18)^2 + 1 = 32$

10. $(f \circ g \circ h)(x)$

$f(g(\underline{h(x)}))$

$g(h(x)) = g(4x - 5) = 4x - 5 + 3$

$g(4x - 5) = 4x - 2$

$f(4x - 2) = (4x - 2)^2 + 1$

$16x^2 - 16x + 4 + 1$

$\boxed{f(g(h(x)) = 16x^2 - 16x + 5}$

11. $(f + g + h)(3)$

$f(3) + g(3) + h(3)$

$f(3) = 3^2 + 1 = 10$

$g(3) = 3 + 3 = 6$

$h(3) = 4(3) - 5 = 7$

$\boxed{(f + g + h)(3) = 23}$

12. $(fgh)(x)$

$f(x) \cdot g(x) \cdot h(x)$

$(x^2 + 1)(x + 3)(4x - 5)$

$(x^3 + 3x^2 + x + 3)(4x - 5)$

$4x^4 - 5x^3 + 12x^3 - 15x^2 + 4x^2$

$-5x + 12x - 1$

$\boxed{4x^4 + 7x^3 - 11x^2 + 7x - 15}$

13. $(f \circ h \circ h)(2)$

$f(h(\underline{h(2)}))$

$h(2) = 4(2) - 5 = 3$

$h(3) = 4(3) - 5 = 7$

$f(7) = 7^2 + 1 = 50$

$\boxed{f(h(h(2))) = 50}$

14. $(f + g + h)(x)$

$f(x) + g(x) + h(x)$

$x^2 + 1 + x + 3 + 4x - 5$

$x^2 + 5x - 1$

$\boxed{(f + g + h)(x) = x^2 + 5x - 1}$

15. $(g \cdot g \cdot f)(x)$ ◯

$g(x) \cdot g(x) \cdot f(x)$

$(x + 3)(x + 3)(x^2 + 1)$

$(x^2 + 6x + 9)(x^2 + 1)$

$x^4 + x^2 + 6x^3 + 6x + 9x^2$

$+ 9$

$\boxed{x^4 + 6x^3 + 10x^2 + 6x + 9}$

16. $\left(\dfrac{fg}{h}\right)(3)$

$\dfrac{f(3) \cdot g(3)}{h(3)} = \dfrac{60}{7}$

$f(3) = 10$

$g(3) = 6$

$h(3) = 7$

$\dfrac{10 \cdot 6}{7} = \dfrac{60}{7}$

17. $(f \circ g \circ f \circ g \circ h \circ h)(1)$

$\boxed{f(g(f(g(h(h(\underline{1})))))) = 1601}$

$h(1) = 4(1) - 5 = -1$

$h(-1) = 4(-1) - 5 = -9$

$g(-9) = -9 + 3 = -6$

$f(-6) = (-6)^2 + 1 = 37$

$g(37) = 37 + 3 = 40$

$f(40) = (40)^2 + 1 = 1601$

◯